In *Encyclopedia of Nonlinear Science* (Ed. A. Scott), pp. 885-887.
Routledge, New York and London (2005).

a compactified dimension of radius R is replaced by another one of radius $1/R$.

In recent years, much research emphasis has concentrated on higher-dimensional objects contained in string and M-theory, so-called D-branes. Open strings can end and start on D-branes. Remember that in electromagnetism there are point charges, which are the sources of the electromagnetic field. D-branes are in a sense higher-dimensional generalizations of these point charges.

As already mentioned, to proceed to our four-dimensional world, six of the dimensions of superstring theory (or 7 of those of M-theory) have to be compactified. The way this should be done is unclear. One needs to compactify on special types of compact manifolds, so-called Calabi–Yau manifolds, to preserve what is generally believed to be the right amount of supersymmetry. The compactification step is clearly necessary for string theory to make contact with the real world, but almost all relevant low energy predictions depend on the numerous possibilities of how to compactify. This severely undermines the predictive power of string theories.

In principle, the ultimate theory should predict everything we want to know about particle physics, such as the values of the coupling constants of the four different forces or the masses of all fermions and bosons. In string theories, the numerical values of gauge couplings (e.g. the fine structure constant α_{el}, which is about $\frac{1}{137}$ at low energies) can be related to vacuum expectations of a scalar field contained in string theories, the dilaton field. But this vacuum expectation depends in an unknown way on second quantization effects and has not been predicted so far. As 't Hooft puts it in his book ('t Hooft, 1997), string theory, at least in its present stage, has similarities with a very uncomplete piece of furniture: "Imagine that I give you a chair while explaining that the legs are still missing, and that the seat, back, and armrest will perhaps be delivered soon; whatever I did give you, can I still call it a chair?"

While typical equations for (free) strings, such as Equation (1), are linear, a recent proposal is to amend ordinary strings (evolving in a regular way) by nonlinear versions of strings, so-called chaotic strings (Beck, 2002). These evolve in a deterministic chaotic way. In this approach, each ordinary string is shadowed by a chaotic string, which yields the "noise" for second quantization via the so-called stochastic quantization method. Mathematically, chaotic strings consist of one-dimensionally coupled Tchebyscheff maps, a very nonlinear and strongly self-interacting theory, which describes a kind of "turbulent quantum state" on a small (quantum gravity) scale. It turns out that the vacuum energy of chaotic strings is minimized for observed standard model parameters; that is, in this extended approach to second quantization, concrete predictions for vacuum expectations of dilaton-like fields and hence on masses and coupling constants can be given.

CHRISTIAN BECK

See also **General relativity; Particles and antiparticles; Quantum field theory**

Further Reading

Beck, C. 2002. *Spatio-temporal Chaos and Vacuum Fluctuations of Quantized Fields*, Singapore: World Scientific

Davies, P.C.W. & Brown, J. 1988. *Superstrings: A Theory of Everything?* Cambridge and New York: Cambridge University Press

Gauntlett, J.P. 1998. M-theory: strings, duality and branes. *Contemporary Physics*, 39: 317–328

Green, M.B., Schwarz, J.H. & Witten, E. 1987. *Superstring Theory*, 2 vols, Cambridge and New York: Cambridge University Press

't Hooft, G. 1997. *In Search of the Ultimate Building Blocks*, Cambridge and New York: Cambridge University Press

Kaku, M. 1999. *Introduction to Superstrings and M-theory*, Berlin and New York: Springer

Polchinski, J. 1998. *String Theory*, 2 vols, Cambridge and New York: Cambridge University Press

STRONG COLLAPSE

See **Development of singularities**

STRUCTURAL COMPLEXITY

By *structural complexity* we mean the study of morphologically complex patterns associated with many physical entities of dynamical systems, using methods based on algebraic, geometric, and topological information (Ricca, 2001). In this respect, structural complexity offers an alternative and complementary route to study complex systems, based on morphological characteristics rather than the governing rules generating complexity. Structural complexity arises from spontaneous growth and self-organization of interacting components present in the bulk of physical systems, such as in turbulence, in biological and chemical reactions, in growth phenomena, across scales of space, time, and organizational complexity (Badii & Politi, 1997). Much of our understanding comes from observations of natural phenomena, from simulations and computational visualizations. Mathematical methods are used to extract and synthesize the relevant information, and to explore possible relationships between morphological complexity, energy localization, and other physical properties.

Dynamical systems are often characterized by the emergence and interaction of *coherent structures* (Nicolis & Prigogine, 1998; Scott, 2003). These latter are produced through filamentation mechanisms and space-time localization of characteristic physical properties. Examples of coherent filamentary structures may be as disparate as DNA macromolecules, synthetic polymer chains, nerve fibers, actin filaments, vortex filaments, magnetic flux tubes, or massive cosmic

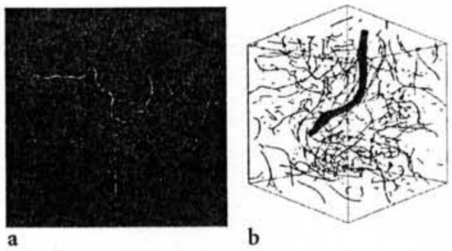

Figure 1. Computational field visualization of a pattern of vortex filaments produced by turbulent flow in a numerical box: (a) low-pressure line and (b) iso-pressure surface, obtained by numerical extraction techniques (Courtesy of S. Kida, Theoretical Division, National Center for Fusion Research, Toki, Japan).

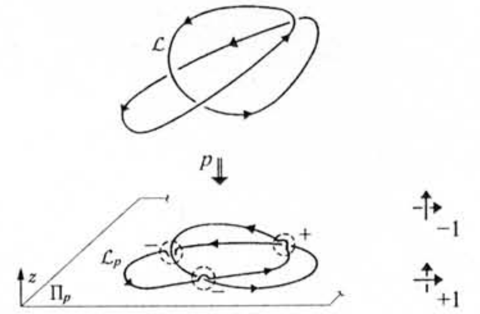

Figure 2. Example of analysis of an oriented space curve $\mathcal{L}$, based on crossing number information of the projected curve $\mathcal{L}_p$: we can calculate the total number of un-signed crossings (3, in the example shown in the figure) and, according to the sign convention shown on the right-hand side of the diagram, the algebraic sum of the $\pm$ signs ($-1-1+1=-1$, in the example shown). By averaging these two quantities over all directions p of projections, we obtain estimates of, respectively, the average crossing number and the writhing number of $\mathcal{L}$.

strings. Dislocations and discontinuities in mesoscopic and solid-state physics, fronts and cell membranes in chemistry, fluid dynamics, and biology provide other examples of complex structuring. The formation and continuous re-arrangement of coherent structures are due to local interactions and recombination, governed by specific dynamics given by nonlinear differential equations $\dot{x}=F(\mathbf{x},\lambda)$, with a nonlinear function F of the vector $x=(x_1,\ldots,x_n)$, which depends on an external control parameter λ. At certain critical values, this parameter causes phase transitions and pattern changes (Levin, 2002). *Pattern recognition* techniques aim at providing mathematical methods for diagnostics of the morphological complexity of the emerging pattern, regardless of the governing fundamental physical mechanism that is responsible for complexity.

One of the great benefits of modern capability to perform complex three-dimensional simulations is the possibility to effectively analyze and visualize huge sets of three-dimensional data. Direct volume rendering, iso-surface extraction, integral convolution, and many other computational techniques allow sophisticated multi-field visualizations of complex physical patterns (Johnson et al., 2001). Progress in *visiometrics*, based on the exploitation of such techniques for an accurate identification and representation of complex three-dimensional structures, provides powerful tools and an almost inexhaustible source of information for analysis of structural complexity (see Figure 1; also see **Visiometrics**).

Physical as well as computational domains are decomposed into *tropicity* domains, defined by the characterizing properties of nulls, tubes, sheets, and blobs. These are reference sets defined by the reference physical field distribution, that allow direct measurement of the degree of *tubeness*, *sheetness*, and *bulkiness* of the coherent set by analyzing the aspect ratio of the distribution.

Algebraic, geometric, and topological measures are used to quantify structural complexity. In case of high filamentation, the original network of filaments is reduced (by appropriate threshold filtering) to a tangle $\mathcal{T}$ of space curves (representing the filament axes), hence reducing the problem to the analysis of the mutual positioning of the system of curves in space. By using methods based on crossing number information, we can evaluate the *average crossing number* $\bar{C}$ of the tangle that provides a fundamental algebraic measure of structural complexity. This quantity is defined by the formula

$$\bar{C}=\sum_{\mathcal{L}_i,\mathcal{L}_j\in\mathcal{T}}\bar{C}_{ij}, \tag{1}$$

where

$$\bar{C}_{ij}=\left\langle\sum_{r\in\mathcal{L}_i\sqcap\mathcal{L}_j}|\varepsilon_r|\right\rangle,\quad \varepsilon_r=\pm1 \tag{2}$$

denotes the un-signed crossings between the curve $\mathcal{L}_i$ and $\mathcal{L}_j$ of the tangle $\mathcal{T}$, and summation is made over all the crossings resulting from $\mathcal{L}_i\sqcap\mathcal{L}_j$; the average number of crossings is then evaluated by projecting the tangle onto projection planes; here the angular brackets denote averaging over all directions of projections (in the simple case of one oriented space curve, Figure 2 shows an application of this analysis). Geometric measures are based on integral measures of curvature and torsion of the reference curves and surfaces; for tangles of curves, these include *total curvature*, *average twist*, and *writhing number* which give information on the amount of coiling and entwinement of the corresponding filaments. Topological information comes from different types of measures: number of *nulls* (zero-value field singularities), *linking numbers* for open or closed curves, winding numbers and invariants associated with handle-body decomposition of reference sets are all calculable quantities. Further information may come from analysis of cell-decomposition of the network. In particular situations, when a prevailing type of geometry emerges, ad hoc analysis is used; for example, spiral spectrum analysis can be employed for scroll-wave reactions in Belousov–Zhabotinsky-type chemical systems and braid theory can be applied to study dynamics of particulates, fluid mixing, trailing vortices, astrophysical plasma loops, and tangle diagrams for polymers. Other information may come from study of dynamical systems, where we have information, for example, on Lyapunov exponents, topological entropy, multifractal properties, and topological scaling (see, for example, de Gennes, 1979; Jensen, 1998), whereas artificial intelligence provides us tools based on algorithmic complexity for neural-type networks. Measures of structural complexity are implemented computationally as time-dependent variables that change with the evolution of the physical system.

Collected information on structural complexity is analyzed against physical information. Since the evolution of any physical system is driven by variational principles that take account of energy and entropy redistributions, information on structural complexity may disclose some useful information on the physics and dynamics of the process. It is known that geometric properties strongly influence the dynamics: curvature forces (such as bending force in elastic rods or surface tension in liquid films) cause telephone cords to coil-up and foam bubbles to coalesce. We, therefore, expect relationships between geometry, dynamics, and ultimately, energy. Possible relationships between energy, entropy, and structural complexity in terms of algebraic information have, however, a more subtle rationale: network restructuring is due to local interactions and recombinations that are induced by acting potentials. These are associated with localized coherent structures, acting either on the strands of neighboring filaments (as in the case of reconnection of vortex tubes) or on the surface elements of sheet discontinuities (as in the formation of elastic films). In this context, local interaction of neighboring structures, resulting in the emergence of apparent crossings (possibly weighted by some distribution function) is a measure of the localization of internal energy. Hence, the resulting growth rate of the average crossing number provides an estimate of energy and entropy variations in the system. In absence of dissipation, topology is conserved; however, even in fully dissipative systems, topological information provides indications to detect preferred paths of energy depletion. In general, if χ denotes a measure (or a family of measures) of structural complexity, and E the energy of the system, we may expect *energy-complexity relations* of type $E = G(\chi)$, where G is a nonlinear function, where the nonlinearity is prescribed by the growth rate exponents of the physical process (see, e.g., Barenghi et al., 2001). Algebraic, geometric, and topological measures of structural complexity are, therefore, not only useful for computational implementation of diagnostics and pattern recognition, but they can also provide new tools to investigate localization and transfer of energy in complex physical systems.

RENZO L. RICCA

See also **Algorithmic complexity; Filamentation; Knot theory; Pattern formation; Topological defects; Vortex dynamics of fluids**

Further Reading

Badii, R. & Politi, A. 1997. *Complexity*, Cambridge and New York: Cambridge University Press

Barenghi, C., Ricca, R.L. & Samuels, D.C. 2001. How tangled is a tangle? *Physica* D, 157: 197–206

de Gennes, P.G. 1979. *Scaling Concepts in Polymer Physics*, Ithaca, New York: Cornell University Press

Jensen, M.H. 1998. *Self-Organized Criticality*, Cambridge and New York: Cambridge University Press

Johnson, C.R., Livnat, Y., Zhukov, L., Hart, D. & Kindlmann, G. 2001. Computational field visualization. In *Mathematics Unlimited – 2001 and Beyond*, edited by B. Engquist & W. Schmid, New York: Springer

Levin, S.A. 2002. Complex adaptive systems: exploring the known, the unknown and the unknowable. *Bulletin of the American Mathematical Society*, 40(1): 3–19

Nicolis, G. & Prigogine, I. 1998. *Exploring Complexity*, New York: W.H. Freeman

Ricca, R.L. (editor) 2001. *An Introduction to the Geometry and Topology of Fluid Flows*, Dordrecht: Kluwer

Scott, A. 2003. *Nonlinear Science: Emergence and Dynamics of Coherent Structures*, Oxford and New York: Oxford University Press

SUBHARMONIC GENERATION

See **Harmonic generation**

SUPERCONDUCTING QUANTUM INTERFERENCE DEVICE

The Superconducting QUantum Interference Device (SQUID) is the most sensitive detector for magnetic flux, Φ. It relies on the fundamental concept of flux quantization in a multiply connected superconductor, for example, a ring (torus). Flux quantization is a clear demonstration of the quantum mechanical properties of a superconductor in which the conduction electrons form (Cooper) pairs and condense in the ground state: the so-called "Bose condensation." Below the transition temperature, T_c, all the bosonic pairs can be described by a single macroscopic many-body wave function

$$\Psi(\mathbf{r}) = [n_s(\mathbf{r})]^{1/2} e^{i\theta(\mathbf{r})}, \qquad (1)$$

where n_s is the density of Cooper pairs. The phase $\theta(\mathbf{r})$ is a scalar function of the position. For small fields one may ignore the spatial dependence of n_s.

A direct consequence of the condensation is that for $T < T_c$ a superconductor exhibits zero electrical resistance and that magnetic fields are expelled from its interior. However, this diamagnetism (Meissner effect)