Flow induction and transport suppression due to helicity, with its implication to subgrid-scale modelling of turbulence

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Summary

Effect of helicity in inhomogeneous turbulence is investigated with special reference to spontaneous large-scale flow generation and transport suppression. With the aid of the multiple-scale perturbative renormalization analysis of non-mirrorsymmetric turbulence, a Reynolds stress expression is derived, which includes inhomogeneous turbulent helicity as the coupling coefficient for the mean absolute vorticity. This inhomogeneous helicity effect may counterbalance the enhanced transport due to turbulence such as eddy viscosity, leading to the possibility of transport suppression and flow generation. Examples of helicity effect are presented. Also a subgrid-scale (SGS) turbulence model with structural effects incorporated through helicity is proposed.

1 Closure analysis of helicity effect

Helicity is an inviscid topological invariant of hydrodynamics. Its turbulent local density (velocity–vorticity correlation) represents the breakage of mirrorsymmetry in turbulence. The Reynolds stress of inhomogeneous turbulence is theoretically calculated in a propagator renormalization method with a multiple-scale analysis. With the aid of derivative expansion:

$$\left\langle u'^{\alpha} u'^{\beta} \right\rangle = \left\langle {u'_{\mathrm{B}}}^{\alpha} {u'_{\mathrm{B}}}^{\beta} \right\rangle + \left\langle {u'_{\mathrm{B}}}^{\alpha} {u'_{01}}^{\beta} \right\rangle + \left\langle {u'_{01}}^{\alpha} {u'_{\mathrm{B}}}^{\beta} \right\rangle + \cdots + \left\langle {u'_{\mathrm{B}}}^{\alpha} {u'_{10}}^{\beta} \right\rangle + \left\langle {u'_{10}}^{\alpha} {u'_{\mathrm{B}}}^{\beta} \right\rangle + \cdots . \tag{1}$$

Homogeneity and isotropy with non-mirrorsymmetry are assumed for the lowest-order field $\mathbf{u}'_{\mathrm{B}}$:

$$\begin{aligned} &\langle {u'_{\mathrm{B}}}^{\alpha}(\mathbf{k},\mathbf{X};\tau,T) {u'_{\mathrm{B}}}^{\beta}(\mathbf{k}',\mathbf{X};\tau',T) \rangle / \delta(\mathbf{k}+\mathbf{k}') \\ &\quad = D^{\alpha\beta}(\mathbf{k}) Q_{\mathrm{B}}(k,\mathbf{X};\tau,\tau',T) + (ik^{a}/2k^{2}) \epsilon^{\alpha\beta a} H_{\mathrm{B}}(k,\mathbf{X};\tau',T). \end{aligned} \tag{2}$$

Effects of inhomogeneity and anisotropy systematically arise from the higher-order terms in Eq. (1). The traceless part (denoted by $[\ldots]_{\mathrm{D}}$) of the Reynolds stress is derived as[1]

$$\langle u'^{\alpha} u'^{\beta} \rangle_{\mathrm{D}} = -\nu_{\mathrm{T}} \mathcal{S}^{\alpha\beta} + [\Gamma^{\alpha}(\Omega^{\beta} + 2\omega_{\mathrm{F}}^{\beta}) + \Gamma^{\beta}(\Omega^{\alpha} + 2\omega_{\mathrm{F}}^{\alpha})]_{\mathrm{D}}, \tag{3}$$

where the transport coefficients are given as

$$\nu_{\mathrm{T}} = \frac{7}{15} \int \mathrm{d}\mathbf{k} \int_{-\infty}^{t} d\tau_1 \; G(k;\tau,\tau_1) Q(k;\tau,\tau_1), \quad \mathbf{\Gamma} = \frac{1}{30} \int k^{-2} \mathrm{d}\mathbf{k} \int_{-\infty}^{t} d\tau_1 \; G(k;\tau,\tau_1) \nabla H(k;\tau,\tau_1) \tag{4}$$

(G: Green's function of turbulence, Q: energy spectral function, H: helicity spectral functions). The first term in Eq. (3) is the effect of the turbulent intensity [1st of Eq. (4)] coupled with the mean velocity strain, usually known as the eddy viscosity, which enhances the effective momentum transport. The second term represents the effects of the helicity-gradient [2nd of Eq. (4)] coupled with the absolute vorticity, which may counterbalance the eddy-viscosity effect.

2 Transport suppression and flow induction by helicity

Transport suppression - swirling flow Using the analytical expression of the Reynolds stress [Eq. (3)], a turbulence model with the helicity effect is constructed. In addition to the turbulent energy and its dissipation rate, the turbulent helicity is considered as a measure of the transport suppression. This model is reduced to the usual eddy-viscosity model for vanishing turbulent helicity, and is a natural extension of the usual eddy-viscosity turbulence model. The model is applied to a turbulent swirling flow (Fig. 1). It is shown that flow properties such as the deceleration at the central axis, etc. are reproduced, which can not be obtained from the standard model.

Flow generation - vortex dynamo In addition to the transport suppression, this inhomogeneous helicity effect may play an essential role in vortex dynamo, global vorticity generation mechanisms in turbulence. With the aid of direct numerical simulations (DNSs) of a rotating turbulence with inhomogeneous helicity externally imposed, it is shown that a large-scale flow can be induced from inhomogeneous turbulent helicity (Fig. 2)[2]. Physical origin of such a global flow is also discussed.

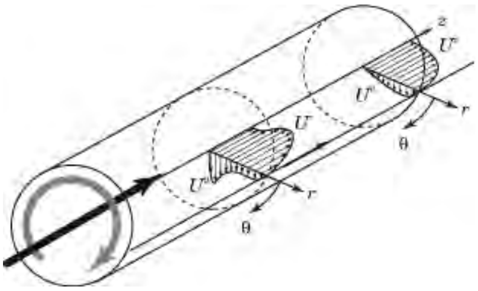

Fig. 1: Schematic turbulent swirling flow. Axial and circumferrential velocity profiles.

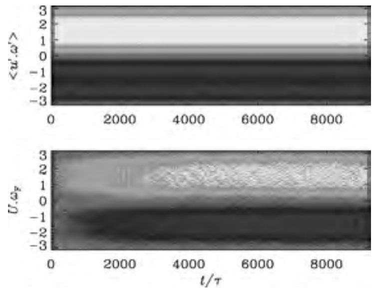

Fig. 2: Temporal evolutions of the turbulent helicity (upper) and mean axial velocity (lower).

3 SGS modelling with helicity

It has been pointed out that in order to improve large-eddy simulation (LES) performance, structural information of turbulence should be implemented into the SGS modelling. On the analogy of Eq. (3), we construct the SGS model, where effects of structure including streamwise vorticity are incorporated into the SGS stress through the SGS helicity as

$$[\overline{u^\alpha u^\beta} - \overline{u}^\alpha \overline{u}^\beta]_{\rm D} = -\nu_{\rm SGS} \overline{s}^{\alpha\beta} + [\gamma^\alpha \overline{\omega}_*^\beta + \gamma^\beta \overline{\omega}_*^\alpha]_{\rm D}, \tag{5}$$

where $\overline{f}$ denotes the filtered or grid-scale (GS) quantity of f ($\overline{s}$: GS velocity strain, $\overline{\omega}_*$: GS absolute vorticity). Several levels of the SGS model are proposed. The most straightforward one is to use Eq. (5) in the GS momentum equation with the transport equations of the SGS energy and helicity. At the simplest zero equation model, where no additional equations are solved other than the GS momentum equation as in the Smagorinsky model, we have to estimate the SGS helicity, as well as the SGS energy, in terms of the GS quantities. By assuming the local equilibrium between the production and dissipation of the SGS helicity, a candidate for the SGS helicity estimate is proposed. It is expected that the problem of constant adjustment (too much turbulent transport) associated with the usual Smagorinsky-type model is alleviated with this helicity SGS model.

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References

[1] Yokoi, N. & Yoshizawa, A. (1993) Statistical analysis of the effects of helicity in inhomogeneous turbulence. *Phys. Fluids* 1993 A**5**, 464–477.

[2] Yokoi, N. & Brandenburg, A. (2016) Large-scale flow generation by inhomogeneous helicity. *Phys. Rev. E* 2016, submitted.