Hybrid convection in astrophysical discs

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Summary

We examine the local convective stability of hydrodynamic discs with radial and vertical stratification in the presence of thermal diffusion (or relaxation). We find that in the regime of comparable radial and vertical Richardson numbers (i.e. $|Ri_x| \sim |Ri_z|$) and wavenumbers (i.e. $|k_x| \sim |k_z|$) the disc becomes unstable, even in the presence of radial and vertical stratifications with $Ri_x > 0$ and $Ri_z > 0$. The origin of this resides in an hybrid radial-vertical Richardson number. We will present an equilibrium profile with temperature depending on the radial and vertical coordinates and with $Ri_z > 0$ for which this destabilization mechanism occurs in the proximity of the midplane.

Astrophysical discs composed of ionized gases accrete toward a central object because subject to the magnetorotational instability (MRI), which induces transport of angular momentum outwardly. Extended zones of protoplanetary discs are, however, scarcely ionized. In these regions the MRI drive is absent and other instabilities of hydrodynamical nature must be active in order to explain the process of planet formation. Convection is a possible candidate.

Urpin (2003) investigated generally the local stability of a fully stratified and thermally diffusing disc in the presence of radial and vertical convection. Assuming exponential time dependence of the type e^{st} for the perturbations, the relevant axisymmetric dispersion relation ([1], [2]) in non-dimensional form reads

$$s^3 + k^2 Pe^{-1} s^2 + s\left[\frac{k_z^2}{k^2}[2(2-\tilde{q}) + Ri_x] - \frac{k_x k_z}{k^2}\Big(2A_z + \frac{L_{Sx}}{L_{Sz}} Ri_x + \frac{L_{Sz}}{L_{Sx}} Ri_z\Big) + \frac{Ri_z k_x^2}{k^2}\right] + Pe^{-1}[2k_z^2(2-\tilde{q}) - 2k_x k_z A_z] = 0, \quad (1)$$

where Pe is the Peclet number, L_S represents the entropy length scale, $\tilde{q}$ and A_z are the radial and vertical velocity shear coefficients respectively. Ri is the Richardson number and k the wavenumber, x and z represent the radial and vertical coordinates respectively. From this equation we can obtain the condition for convective instability which reads

$$\left[\frac{k_z^2}{k^2} Ri_x - \frac{k_x k_z}{k^2}\left(\frac{L_{Sx}}{L_{Sz}} Ri_x + \frac{L_{Sz}}{L_{Sx}} Ri_z\right) + \frac{Ri_z k_x^2}{k^2}\right] < 0 \quad (2)$$

Here we are particularly interested in studying equation (2) when radial and vertical stratifications are comparable (i.e. $|Ri_z| \sim |Ri_x|$) and for wavenumbers $|k_z| \sim |k_x|$. If k_x and k_z have the same sign equation (2) reads:

$$Ri_x\Big(1 - \frac{L_{Sx}}{L_{Sz}}\Big) + Ri_z\Big(1 - \frac{L_{Sz}}{L_{Sx}}\Big) < 0. \quad (3)$$

When $Ri_x > 0$ and $Ri_z > 0$ the disc is unstable if $L_{Sx}/L_{Sz} > 0$. The critical term driving the disc to instability is the hybrid Richardson number

$$Ri_{xz} = \frac{L_{Sx}}{L_{Sz}} Ri_x + \frac{L_{Sz}}{L_{Sx}} Ri_z. \quad (4)$$

This points at the fact that when discs are fully stratified, instability drives arise not only from purely radial and vertical gradients but as well from mixed radial-vertical ones.
At the presentation we will discuss some concrete examples of equilibrium profiles where this hybrid convective instability occurs with growth rates in some regions of the same order of the local orbital frequency.

References

[1] Urpin V. (2003) A comparison study of the vertical and magnetic shear instabilities in accretion discs. *Astronomy & Astrophysics* 2003 **404**, 397-403.

[2] Volponi, F. (2014) Non-axisymmetric vertical shear and convective instabilities as a mechanism of angular momentum transport. *Monthly Notices of the Royal Astronomical Society* 2014 **441** 813-820.