# Periodic orbits of analytic Euler fields

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**Summary**

On any compact analytic 3-manifold without boundary which is not a torus bundle over the circle, any analytic vector field with no zeros and which is solution to the stationary Euler equations has a periodic orbit. This result can be traced back to the works of A. Rechtman [2] and K. Cieliebak and E. Volkov [1]. In this talk we will present an alternative proof of this fact, using ideas from contact and differential topology.

The existence of closed orbits is one of the most natural qualitative questions on the behaviour of a dynamical system. We want to address it in the context of stationary Euler flows.

A stationary Euler flow on a three dimensional Riemannian manifold $(M, g)$ is described by a velocity field $u(x)$ which satisfies the stationary Euler equations

$$i_\omega i_u \mu = dB\,, \qquad d i_u \mu = 0$$

where $\mu$ is the volume form on $(M, g)$, $B(x)$ is the so-called Bernoulli function and $\omega := curl\ u$ is the vorticity field.

In this talk we are to present a proof of the following:

**Theorem** Let (M, g, $\mu$) be a three dimensional compact, analytic, Riemannian 3-manifold without boundary (where the volume form $\mu$ is not necessarily the one given by the metric $g$). Let $u$ be a nowhere vanishing $C^\omega$ vector field verifying the stationary Euler equations

$$i_\omega i_u \mu = dB\,, \qquad d i_u \mu = 0$$

Then, if $M$ is not a torus bundle over $\mathbb{S}^1$, $u$ has a closed orbit.

If, instead of being analytic, the field is just $C^\infty$, the question remains completely open: the stratified structure of critical sets of analytic functions is key to the proof.

## References

[1] K. Cieliebak and E. Volkov, A note on the stationary Euler equations of hydrodynamics. Ergodic Theory and Dynamical Systems (2015).

[2] A. Rechtman, Existence of periodic orbits for geodesible vector fields on closed 3-manifolds. Ergodic Theory and Dynamical Systems 30(6) (2010) 1817-1841.