Chiral anomalies, helicity and information geometry

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Summary

Fluid helicity is an important observable that captures topological properties of hydrodynamics. It naturally emerges in the context of parity-breaking fluids with knotted vortex lines. If the fluid constituents exhibit quantum anomalies the topological nature of fluid helicity can be elucidated using microscopic physics. In this case the helicity is given by a polynomial function of temperature and chiral chemical potential and completely fixed by the anomalies. We explain this relation and address the question of instabilities of such fluids using methods of information geometry. We introduce the metric on a parameter space and show that a non-zero vorticity leads to a curvature on the statistical manifold. We calculate the curvature invariant and analyze its divergences, which contain the information about phase transitions of the system. The transition points are universal and expressed in terms of ratios of anomaly coefficients.

Hydrodynamics is an effective field theory which can be used to describe many different physical systems. One of the recent applications of the theory of relativistic hydrodynamics is the study of the evolution chiral fluids. The relativistic hydrodynamic equations have been proposed many years ago [1]; such equations describe the dynamics of an interacting relativistic theory at large distance and time scales. The hydrodynamic variables are the local velocity $u^\mu(x)$ (satisfying $u^2 = -1$), the local temperature $T(x)$ and chemical potential(s) $\mu^a(x)$, where the index a numerates the conserved charges. The hydrodynamic equations govern the time evolution of these variables; they have the form of the conservation laws $\partial_\mu T^{\mu\nu} = 0$, $\partial_\mu j^{a\mu} = 0$, supplemented by the constitutive equations which express $T^{\mu\nu}$ and $j^{a\mu}$ in terms of u^μ, T, and μ^a. These equations are the relativistic generalization of the Navier-Stokes equations.

One feature of relativistic quantum field theory that does not have direct counterpart in non-relativistic physics is the presence of triangle anomalies [2, 3]. For currents associated with global symmetries, the anomalies do not destroy current conservations, but are reflected in the three-point functions of the currents. When the theory is put in external background gauge fields coupled to the currents, some of the currents will no longer be conserved.

In the simplest case when there is one U(1) current with a U(1)^3 anomaly. We consider only global currents that are not coupled to dynamical gauge fields, and assume the associated symmetries are not spontaneously broken. The constitutive equation for the conserved current j^μ must contain an additional term proportional to the vorticity.

$$j^\mu = nu^\mu - \sigma T(g^{\mu\nu} + u^\mu u^\nu)\partial_\nu \left(\frac{\mu}{T}\right) + \xi\omega^\mu, \tag{01}$$

$$\omega^\mu = \frac{1}{2}\epsilon^{\mu\nu\lambda\rho}u_\nu\partial_\lambda u_\rho, \tag{02}$$

where n is the charge density, σ is the conductivity, and ξ is the new kinetic coefficient.

Even in a parity-invariant theory, the vorticity-induced current $\xi\omega^\mu$ is allowed by symmetries if, e.g., j^μ is a chiral current. This term contains only one spatial derivative, and its effect is as important as those of viscosity or diffusion. Before very recently, this term has been completely overlooked. In fact, if one follows the standard textbook derivation, the new term seems to be disallowed by the existence of an entropy current with manifestly positive divergence, required by the second law of thermodynamics.

In this presentation we will show that this new term is not only allowed, but is required by anomalies. Moreover, the parity-odd kinetic coefficient ξ is completely determined by the anomaly coefficient C, defined through the divergence of the gauge-invariant current, $\partial_\mu j^\mu = -\frac{1}{8}C\epsilon^{\mu\nu\alpha\beta}F_{\mu\nu}F_{\alpha\beta}$, and the equation of state,

$$\xi = C\left(\mu^2 - \frac{2}{3}\frac{\mu^3 n}{\epsilon + P}\right), \tag{03}$$

where ϵ and P are the energy density and pressure. The key ingredient required to derive such transport is the second law of thermodynamics. The entropy current has to be modified to cancel the unwanted terms containing the Levi-Civita symbol which can in principle violate the second law. Therefore the effect is non-dissipative - it does not contribute to the entropy production. As a result it is convenient to formulate it in terms of partition functions [5, 6]. Such a formulation sheds new light on the quantum origin of the transport and allows for generalizations e.g. include gravitational anomalies [7]. The topological nature of anomalies manifests itself through the anomaly polynomials $\mathcal{P}_{anom}(F, R)$. They are functions of gauge field strength and curvature, which represent a compact way to describe anomalies. It was argued in [5, 6] that the anomaly induced transport can be described by a closely related polynomial object $\mathfrak{F}^{\omega}_{anom}(\mu, T)$, provided we do the the following substitution

$$\mathfrak{F}^{\omega}_{anom} = \mathcal{P}_{anom}\left(F \mapsto \mu\ ,\ p_1(R) \mapsto -\beta^{-2},\ p_{k>1}(R) \mapsto 0\right), \tag{04}$$

where $p_1(R)$ is the first Pontryagin class of space-time curvature. Subsequently the polynomial object $\mathfrak{F}^{\omega}_{anom}$ was connected to the helicity of the thermal state [8].

Partition functions are primary objects in studying field theories and statistical systems. Unfortunately the number of models, for which partition functions are known exactly is very limited and in order to get a nonperturbative answer one usually invokes underlying integrability or supersymmetry of the model. However, most models do not have such a symmetry and the partition function is known only perturbatively in a limited range of the parameter space. Anomalous hydrodynamics is an exception in this respect. The powerful constraints coming from the connection of QFT symmetry breaking and the laws of thermodynamics make the transport nondissipative and fixed purely in terms of field theory data. As a result the partition function of the theory in the hydrodynamic regime can be constructed analytically. This gives as a closed form expression for the Gibbs probability distribution for the anomalous state. The statistical distributions can be viewed as geometrical manifolds, which has a non-zero curvature and possibly nontrivial phase structure if the system is interacting. This represents a new quantitative tool for the study of fluctuation phenomena, known as information geometry [9].

References

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