Conservation of writhe helicity under anti-parallel reconnection

De Witt Sumners

Department of Mathematics, Florida State University, USA

Summary

The helicity of a flux tube can be calculated in terms of writhe and twist contributions. We prove that the writhe is conserved under anti-parallel reconnection. This is joint work with Christian Laing and Renzo Ricca [1].

Reconnection is a fundamental event in many areas of science, from the interaction of vortices in classical and quantum fluids, and magnetic flux tubes in magnetohydrodynamics and plasma physics, to recombination in polymer physics and DNA biology. By using fundamental results in topological fluid mechanics, the helicity of a flux tube can be calculated in terms of writhe and twist contributions. We prove that the writhe is conserved under anti-parallel reconnection. Hence, for a pair of interacting flux tubes of equal flux, if the twist of the reconnected tube is the sum of the original twists of the interacting tubes, then helicity is conserved during reconnection. Thus, any deviation from helicity conservation is entirely due to the intrinsic twist inserted or deleted locally at the reconnection site. This result has important implications for helicity and energy considerations in various physical contexts.

We will discuss the mathematical similarities between reconnection events in biology and physics, and the relationship between iterated reconnection and curve topology. In particular, the minimal reconnection cascade from $(2, 2k+1)$ torus knots to $(2, 2k)$ torus links to the unlink of two unknotted circles observed in DNA site-specific recombination is also observed in fluid vortex reconnections.

Acknowledgements. Financial support from a Simons Foundation Collaboration Grant for Mathematicians is kindly acknowledged.

References

[1] Laing C.E., Ricca R.L. & Sumners D.W. (2015) Conservation of writhe helicity under anti-parallel reconnection. *Nature Scientific Reports* **5** 9224.