

Helical bottleneck effect in 3D homogeneous isotropic turbulence

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Summary

It is generally believed that helicity can play a significant role in turbulent systems, e.g., supporting the generation of large-scale magnetic fields, but its impact on the spectral properties of turbulent flows is practically negligible. We elaborate the phenomenology of isotropic turbulence for a flow with a high relative helicity over a wide range of scales. Various scenarios of turbulent cascades are discussed. We obtain helicity effect which can be interpreted as a quenching of the spectral energy transfer. The effect is demonstrated by high Reynolds number numerical simulations based on a shell model of helical turbulence. The energy in our model is injected at a certain large scale only, whereas the source of helicity is distributed over all scales. In particular, we found that, depending on the parameters of the injection helicity, a “helical bottleneck effect” can appear in the energy spectrum of isotropic turbulence.

1 Motivation

Many years ago, K. Moffatt delivered a verdict on the influence of helicity on spectral properties of turbulent flows [4]:

no matter how strong the level of helicity injection may be at wave-numbers of order k_0 , the relative level of helicity as measured by the dimensionless ratio $H(k)/2kE(k)$ must grow progressively weaker with increasing k ; and when k/k_0 is sufficiently large it may be conjectured (Brissaud et al., 1973) that the helicity has negligible dynamical effect, and is itself convected and diffused in much the same way as a dynamically passive scalar contaminant (Batchelor, 1959).

Thus the mean helicity can play a significant role in turbulent systems, e.g., supporting the generation of large-scale magnetic fields [2], but its impact on the spectral properties of turbulent flows is practically negligible.

We revise the view that helicity is injected into the flow together with energy at the same scale. Theoretically, one can assume that turbulence is excited by a source of energy at the largest scale and an independent source of pure helicity, acting at a certain scale or over all scales in the inertial interval. Then, the helicity spectral flux is not constant anymore and the helicity spectral density can reach significantly higher values and influence the energy cascade. Real physical situations usually are far from ideal, but can be similar to some extent e.g. in rotating convective flows [3].

2 Phenomenology

First, we adopt the basic statement of Kolmogorov’s approach, which claims that in the inertial range at any scale n the energy flux is equal to the dissipation rate, $\Pi_n^E = \varepsilon$. We consider a geometric sequence of scales and corresponding wave numbers $l_n^{-1} \sim k_n \sim \lambda^n$. The energy flux is related to the velocity pulsations v_n at this scale as

$$\Pi_n^E \approx v_n^3 k_n. \quad (1)$$

Second, we follow the decomposition of velocity pulsations in two helical modes [6], $v_n = v_n^+ + v_n^-$, with corresponding energies $E_n^\pm \sim (v_n^\pm)^2$. Then the energy and helicity at the scale n are

$$E_n = E_n^+ + E_n^-, \quad H_n = H_n^+ + H_n^- = k_n(E_n^+ - E_n^-). \quad (2)$$

The energy flux at scale n is decomposed into four terms: $k_n(v_n^+)^3$, $k_n(v_n^-)^3$, $k_n(v_n^+)^2v_n^-$ and $k_n(v_n^-)^2v_n^+$. For the direct cascades of the energy and helicity the contribution of terms $k_n(v_n^\pm)^2v_n^\mp$ is dominant [6].

The relative helicity $H_n^r = H_n/(k_n E_n)$ allows us to link the intensity of two helical modes $v_n^- = v_n^+ \sqrt{(1 - H_n^r)/(1 + H_n^r)} = \xi_n v_n^+$. Then the energy flux provided by local interactions can be estimated as

$$\Pi_n^E \approx k_n(v_n^+)^3 + k_n(v_n^-)^3 = k_n(v_n^+)^3(\xi_n + \xi_n^2). \quad (3)$$

Replacing (1) by (3) we finally obtain $E_n^+ \sim (\varepsilon/(k_n(\xi_n + \xi_n^2)))^{2/3}$. One can express $E_n^- = \xi_n^2 E_n^+$ from (2) and obtain the total energy

$$E_n = E_n^+ + E_n^- \sim (\varepsilon \zeta_n / k_n)^{2/3}, \quad (4)$$

where the dimensionless variable ζ_n

$$\zeta_n = (1 + \xi_n^2)^{3/2} / (\xi_n + \xi_n^2) \quad (5)$$

depends on $H_r(k)$ and defines ‘‘the degree of helical blocking’’ of the spectral energy flux at a given scale. $|H_r(k)|$ characterizes the dominance of some helical modes over others with the opposite sign, i.e. it is the helical part of the energy. Then a new parameter $\delta(k) = 1 - |H_r(k)|$ corresponds to the non-helical part of the energy, which is free of helicity. For the highly helical case $H_n^r \rightarrow \pm 1$, formula (5) has as asymptote $\zeta_n \approx \delta_n^{-1/2}$. The corresponding spectral energy density

$$E(k) \approx \varepsilon^{2/3} k^{-5/3} \delta(k)^{-1/3} \quad (6)$$

is independent of the sign of the injected helicity. Usually for the single-scale forcing of helicity, $H_n^r \propto k^{-1}$ and the parameter δ_n does not differ from unity. One can expect a significant change in the turbulent spectra for highly helical turbulence only. We verify the realizability of the energy spectrum (4) with helical correction (5) by numerical simulation using the helical shell model of turbulence [5].

We interpret our results as a quenching effect on the spectral energy transfer in scales with high relative helicity. The energy should be accumulated and redistributed so that the efficiency of non-linear interactions will be enough to provide the constant energy flux, which is predetermined by the energy injection rate. A similar consequence is observed as a result of the bottleneck phenomenon [1] in the non-helical turbulent cascade when non-local interactions drop out of the spectral energy transfer at the end of the inertial range. We exploit this analogy to name our effect the *helical bottleneck effect*, having in mind the helical mechanism of cascade blocking.

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