Self-generating magnetic knots in Plasma

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Summary

Plasma configurations with magnetic helicity self-organize into knotted magnetic sturctures characterized by magnetic field lines lying on nested toroidal surfaces. The regularity of this structure allows us to construct an analytical approximation of the field using the Hopf map. Detailed Inspection of the high-β self-organized structure occuring in our full-MHD simulations reveals dynamics similar to magnetic confinement devices: magnetic islands on rational surfaces, a balance of pressure and Lorentz forces, and a toroidal depression in the pressure, with a minimum on the magnetic axis of the configuration.

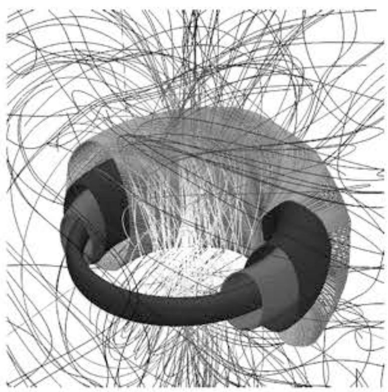

Fig. 1: Magnetic surfaces in a self-organized magnetic structure.

Magnetic helicity, defined as

$$H_{\mathrm{m}} = \int \mathbf{A} \cdot \mathbf{B} \mathrm{d}^3 x \tag{1}$$

is an invariant in an ideal plasma that quantifies the amount of linking of magnetic field lines [2]. In a resistive plasma helicity is only approximately conserved, i.e. the decay of helicity is generally on a much slower timescale than reconfiguration through fluid motion. This allows the fluid to reconfigure a helical magnetic field to a geometry that minimizes magnetic energy, whilst reconnection slowly changes the topology.

We perform full-MHD simulations starting with initial conditions that are unequivocally helical: rings of magnetic flux that are all linked with each other and/or twisted. These fields all pass through a turbulent phase, and reconfigure into organized structures that consist of field lines lying on nested toroidal surfaces. These structures have low dissipation and are quasi-stable. The magnetic forces are balanced by the pressure gradient caused by a toroidal depression in pressure. The nested magnetic organization bears resemblance to the mathematical structure of the Hopf map, a celebrated result in the field of topology. The Hopf map $h : S^3 \to S^2$ is a funtion between spheres such that the fibers of the map (preimages of points on S^2) are all circles lying on nested toroidal surfaces. We show that generalizations of this map [3, 4] can be used to generate an analytical expression that approximates this field.

The different magnetic surfaces in this structure are characterized by a nearly constant rotational transform. The rotational transform of a magnetic surface is a measure introduced in the context of magnetic confinement fusion and is the (asymptotic) ratio of poloidal to toroidal winding number of a field line on that magnetic surfaces. In the self-organzied magnetic structure the rotational transform varies less than 15% from surface to surface in a continuous way. At the surfaces where the value of this rotational transform crosses rational values, only a single (m, n) torus knotted field line remains (with n/m the rational value of the rotational transform on that surface), and nearby field lines form magnetic surfaces around that knotted curve.

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