Asymptotic Solutions for Linear and Nonlinear MHD Systems with a Rapid Jump near a Surface. Dynamics of the Surface of the Jump and Evolution of the Magnetic Field

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Summary

We describe the asymptotic structure of the solution both for linear and nonlinear MHD system with a rapid jump near a 2D-surface. For the linear system we demonstrate the effect of the instantaneous growth of the solution. We also study the weak limit of the solution and the corresponding generalized problem. For the nonlinear system we describe the asymptotic division into different modes, the free boundary problem for the movement of the surface and the equation on the moving surface for the profile of the asymptotic solution. We also study the possibility of the instantaneous growth of the magnetic field. It appears that the growth is possible only in the case of the so-called degenerate Alfvén modes.

Equations of magnetohydrodynamics (MHD equations) describe the motion of the magnetic field in a conducting fluid. This nonlinear system of PDEs consists of the Navier Stokes equations for the velocity field of the fluid and the Maxwell equations for the magnetic field. Usually the viscosity and the resistance of the fluid are small enough and one can study the asymptotic solutions of the system with respect to the corresponding small parameter. This problem was studied in a lot of papers; note that in the linear approximation to the MHD equations the structure of the asymptotics is the subject of the famous dynamo theory (see, e.g., [1], [2], [3], [4], [5], [6], [7], [8], [9]). The main mathematical problem of the theory is whether there exists a velocity field such that the solution (i.e., the magnetic field) grows exponentially as time tends to infinity. Such a growth is one of the popular scenarios of the origin of earths, galaxies and stars magnetic fields from small initial perturbations. The origin of the growth is connected with the chaotic behavior of the trajectories of the velocity field of the fluid. Let us mention several well-known results of the theory.

- In the two-dimensional case, the so-called antidynamo theorem [2], [**?**], [9] states that there is no exponential growth of magnetic field as time tends to infinity.
- In [3], the growth was proved in the artificial example - the induction equation defined on a 3D Riemannian manifold with exponentially unstable geodesics.
- In [8], [5] the resolving operator for the Cauchy problem was studied as well as the behavior of localized solutions on the finite time interval. The explicit formulas show the temporal growth in the ABC flow.
- An alternative effect was studied in [10], [11] (in the linear approximation to the MHD equations also). Namely, we described the instantaneous growth of the magnetic field, induced by the jump of the velocity field of the fluid. In other words, we studied the asymptotics of the solution for the Cauchy problem for the linear induction equation with a rapidly varying velocity field. We assumed that this field has a rapid jump in a small vicinity of the fixed 2D surface. We proved that the solution grows rapidly with respect to the corresponding small parameter, and has a delta-type singularity near the surface of the jump.
- In the recent paper [12] we studied the formal asymptotics for the nonlinear MHD system describing the fields with the rapid jump near a surface. We obtained the free boundary

problem governing the movement of the surface as well as the equations on the moving surface governing the evolution of the magnetic and velocity fields. We proved that the instantaneous growth takes place for the so-called degenerate Alfvén modes only.

Here we present our results concerning the asymptotic solutions for both linear and nonlinear MHD equations. We describe the asymptotic structure of the solution with a rapid jump near the 2D-surface. For the linear system we demonstrate the effect of the instantaneous growth of the solution. We also study the weak limit of the solution and the corresponding generalized problem. For the nonlinear system we describe the asymptotic division into different modes, the free boundary problem for the movement of the surface and the equation on the moving surface for the profile of the asymptotic solution. We also study the possibility of the instantaneous growth of the magnetic field. It appears that the growth is possible only in the case of the so-called degenerate Alfv?en modes; the latter appear if the main term of the magnetic field is tangent to the surface of the jump.

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