

Helical vortices: linear and nonlinear dynamics

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Summary

We present results on the dynamics of helical vortices. In particular we analyse equilibrium states and their linear and nonlinear stability.

The near wake behind helicopter rotors, wind turbines and more generally behind rotating devices are dominated by helical vortices. Investigating their stability properties is a necessary step to predict their dynamics. The base flows considered here are helically symmetric : fields are invariant through combined axial translation of distance Δz and rotation of angle $\theta = \Delta z/L$ around the z -axis, where $2\pi L$ denotes the helix pitch. In order to simulate the evolution of such flows, we use a Navier-Stokes code based on a formulation in which the helical symmetry is enforced into the equations [1]. Basic states are obtained which consist of multiple diffusing helical vortices, for various values of the helical pitch.

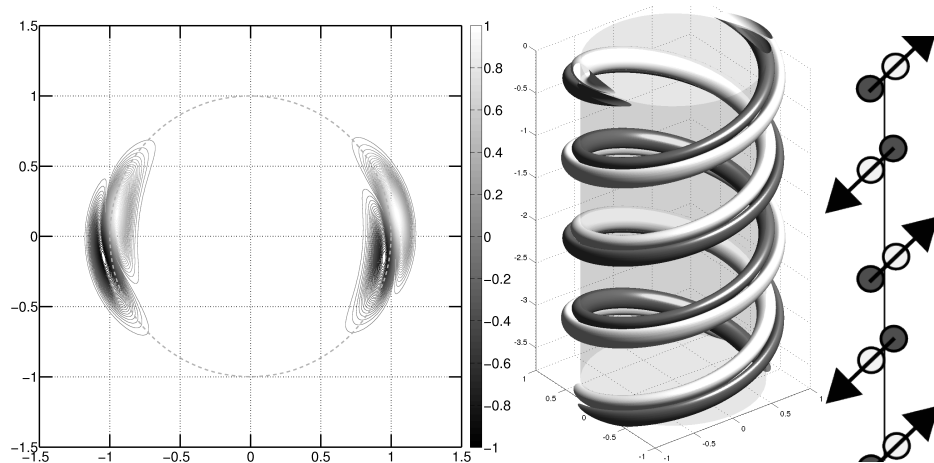


Fig. 1: Linear stability analysis: (a) Structure of the most unstable mode. The base flow corresponds to two helical vortices with pitch $2\pi L = 0.3$ and core size $a_b = 0.09$. The mode is represented by the helical vorticity component of the perturbation field in the horizontal (x, y) -plane by colored contours. (b) Mode in a 3-D representation. The arrows indicate the perturbation action on the base flow. The displacement induced by the mode has two components: one along the radial direction and one along the z direction. On the radial direction, one vortex goes inwards while the other goes outwards. (c) Schematic representation: the structure is analogous to the pairing instability mode for an infinite row of point vortices.

Instabilities in such vortex systems have mainly been studied theoretically [2]–[3] in an inviscid framework for small core size vortices. The aim of the present study is to generalize these works to the viscous framework for arbitrary core sizes and vorticity profiles. We first perform a linear temporal stability analysis of such base flows, using an Arnoldi procedure. Two different codes are used : (i) a linearised version of the nonlinear helical code, (ii) another code which computes the dynamics of *arbitrary perturbations* in the vicinity of a helically symmetric base flow. These codes allows investigating different instability modes : (i) modes having the same helical symmetry as the base flow generalizing the Okulov modes ; (ii) modes with arbitrary wavenumber k along

z generalizing the Widnall modes. In the first case, instabilities are found to be dominated by displacement modes of the type presented in figure 1 for the case of two vortices. In the second case, modes are compared to those observed in recent experimental work [4].

The nonlinear dynamics of a basic flow perturbed with a linear mode of type (i) with an initial weak amplitude is computed. The displacement mode is shown to be responsible for leap-frog dynamics and vortex merging with characteristics depending on the various parameters.

Acknowledgements. Financial support from ANR HELIX project ANR-12-BS09-0023-01 is kindly acknowledged.

References

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