Existence of force-free magnetic fields, and formation of electric current singularities

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Summary

We discuss the existence of smooth equilibria – or as an alternative, the formation of current singularities – in plasmas with low dissipation. We examine in particular magnetic braids, and note that when these braids are 'line-tied' at perfectly conducting planes, smooth equilibria in general exist, though exhibit thin current layers with length scales determined by the field line mapping. In a periodic domain, exact equilibria typically do not exist, while approximate equilibria exhibit thin current layers. We also discuss the case of magnetic null points, at which current singularities develop in response to rather generic perturbations to an initial equilibrium.

1 Background and outline of contribution

The magnetic field is an important driver of plasma dynamics in a wide range of environments. Many phenomena of interest involve explosive release of stored magnetic energy, though our understanding of the mechanisms by which this energy release occurs is in its infancy. Magnetic fields in laboratory and astrophysical environments typically have complex three-dimensional structure, and are often inherently disordered, being characterised by field lines that are tangled with one another in non-trivial ways. This complexity may be measured in various ways, using for example the topological entropy or field line helicity.

Magnetic braids (magnetic flux tubes within which the field lines have some non-trivial winding or linkage) have been used for some time to model coronal loops in the Sun's atmosphere, or *corona*. This was initially a response to Parker's proposed nanoflare heating mechanism [2, 3]. Therein, the corona may be heated to the observed multi-million degree temperatures as a result of turbulent convective motions in the outer layers of the solar interior that tangle or braid the field lines in the corona about one another. A crucial ingredient of this mechanism is the hypothesis that for sufficiently braided magnetic fields no smooth force-free equilibrium exists, but rather the field develops current singularities. Here we present work that challenges this hypothesis by demonstrating the existence of smooth equilibria for a class of magnetic braids [6]. This result holds for the case in which the field is line tied at perfectly conducting plates, as is the case in the solar atmosphere (where the part of the conducting plates in played by the solar surface (or *photosphere*). However, we show that thin current layers must be present in the equilibria, and that these current layers become increasingly thin as the field becomes more tangled. Thus, one may still release energy from the magnetic field by braiding the field sufficiently that these current layers approach the diffusive length-scale. By contrast to the above, when one considers a periodic domain as is relevant for many laboratory devices, smooth equilibria can be shown not to exist, while a field close to equilibrium must contain thin current layers [4].

We subsequently contrast the case of magnetic braids with magnetic fields that contain null points (points in space at which is magnetic field strength falls to exactly zero). In this case, it is shown that a rather generic perturbation of the field leads to an unbounded growth of the electric current, corresponding to a current singularity in the perfectly-conducting limit [1, 5].

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