

Helicity is the only integral invariant of volume-preserving transformations

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Helicity is a remarkable conserved quantity that is fundamental to all the natural phenomena described by a vector field whose evolution is given by volume-preserving transformations. This is the case of the vorticity of an inviscid fluid flow or of the magnetic field of a conducting plasma. The topological nature of the helicity was unveiled by Moffatt, but its relevance goes well beyond that of being a new conservation law. Indeed, the helicity defines an integral invariant under any kind of volume-preserving diffeomorphisms. A well-known open problem is whether any integral invariants exist other than the helicity. We answer this question by showing that, under some mild technical assumptions, the helicity is the only integral invariant. Specifically, given a functional \mathcal{I} defined on exact divergence-free vector fields of class C^1 on a compact 3-manifold that is associated with a well-behaved integral kernel, we prove [1] that \mathcal{I} is invariant under arbitrary volume-preserving diffeomorphisms if and only if it is a function of the helicity.

References

- [1] Enciso, A., Peralta-Salas, D. & Torres de Lizaur, F. (2016) Helicity is the only integral invariant of volume-preserving transformations. *Proc. Natl. Acad. Sci. USA* **113** 2035–2040.