Magnetic Connection Hypersurfaces in Relativistic Magnetohydrodynamics

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Summary

In the single fluid, nonrelativistic, ideal-Magnetohydrodynamic (MHD) plasma description magnetic field lines play a fundamental role by defining dynamically preserved "magnetic connections" between plasma elements. Here we show how the concept of magnetic connection needs to be generalized in the case of a relativistic MHD description where we require covariance under arbitrary Lorentz transformations. This is performed by defining 2-D *magnetic connection hypersurfaces* in the 4-D Minkowski space. This generalization accounts for the loss of simultaneity between spatially separated events in different frames and is expected to provide a powerful insight into the 4-D geometry of electromagnetic fields when $\vec{E} \cdot \vec{B} = 0$.

1 Physical motivation

The dynamics of large scale relativistic plasma configurations plays an important role in our understanding of high energy astrophysical phenomena such as, just to mention a recently discovered one, the flaring of the Crab nebula. Even without including general relativistic effects, as would be the case e.g., in the neighbourhood of a black hole, the phenomena we need to describe involve velocities close to the speed of light and internal energies that can be larger than the electron rest mass energy. With this in mind, several concepts that have been introduced for nonrelativistic plasmas needs to be extended to relativistic regimes. In such a generalization space and time properties are necessarily combined since the basic invariance properties of the matter equations are now given in terms of the Lorentz group of transformation between different reference frames. This is particularly important since, in the presence of very large velocity differences between different parts of the plasma configuration, there may not be a clear way to define on physics grounds a preferred reference frame. In addition, the observer reference frame may move with a relativistic velocity with respect to the plasma under observation and thus observe as simultaneous events that are not simultaneous in the plasma frame.

A number of basic phenomena of nonrelativistic MHD, such as e.g., magnetic reconnection, have been reconsidered in relativistic plasma regimes both in the laboratory and in astrophysics. In particular in the astrophysical context relativistic magnetic reconnection has been considered mostly as a mechanism of energy conversion, usually choosing a preferred frame of reference possibly thought of as an "average comoving frame" i.e. as a frame in which the plasma region under consideration is globally at rest. As mentioned above, such an approach may not be fully unambiguous in situations where very large velocity relativistic variations can be present between different plasma regions, in particular since magnetic fields and electric fields are transformed one into the other when seen in a Lorentz boosted reference frame. Thus an important point in the relativistic extension of the MHD plasma description is to provide a frame independent definition of magnetic reconnection.

Although a clearcut definition of magnetic reconnection is not simple to formulate even for a non relativistic plasma, its common definition is not simply limited to the fact that magnetic energy is converted to kinetic and/or internal plasma energy, but refers to the local violation of the magnetic topology and in particular to the local breaking of the structure of magnetic connections. Magnetic connections are defined by the fundamental property of ideal MHD (see Ref.[1]) that if two plasma elements, moving with plasma in a smooth flow, are connected at time t by a magnetic field line then at any following time there exists a magnetic field line that connects them. Thus in order to define magnetic reconnection in a covariant way we must first obtain a covariant definition of magnetic connections. Again, such a definition is not *a priori* obvious because of two already mentioned related reasons: the distinction between electric and magnetic fields and the very concept of field lines are frame dependent. This point was explicitly addressed in Ref.[2] where it was shown that the covariant formulation of magnetic connections can be restored by means of a *time resetting* projection along the trajectories of the plasma elements. This projection is consistent with the ideal Ohm's law and compensates for the loss of simultaneity in different reference frames between spatially separated events.

2 Mathematical developments

Here we address this same issue and show that the time resetting along the trajectories of the fluid elements introduced in Ref.[2] is essentially equivalent to a redefinition of the geometrical object that we use in order to define magnetic connections. We argue that, while in 3-D (coordinate) space magnetic connections are defined by 1-D curves (field lines), in the 4-D Minkowski space they are defined by 2-D hypersurfaces that are generated by a suitably defined magnetic (space-like) 4-vector field (see Refs.[3, 4]) and by the velocity (time-like) 4-vector field of the plasma.
We show that if the electromagnetic (e.m) field tensor satisfies an ideal Ohm's law, it exhibits special geometrical properties that are simply the consequence of the homogeneous Maxwell's equations and that make it possible to define such 2-D hypersurfaces so that, if in a given frame two plasma elements in 4-D Minkowski space lie on the same 2-D hypersurface, they do so in any other reference frame. We call these 2-D hypersurfaces *Covariant Magnetic Connection Hypersurfaces*. The standard magnetic connections in 3-D space can then be recovered in any chosen reference frame by taking sections of these surfaces at a fixed (in that frame) time. We stress that these 2-D hypersurfaces bear no relation to the 3-D magnetic surfaces of nonrelativistic MHD that, if generalized to 4-D Minkowski space, would involve 3-D "volumes".

We stress that the violation of the ideal Ohm's law leads to a violation of the geometrical properties of the e.m. field tensor that make it possible to define the connection hypersurfaces. Thus in this 4-D framework magnetic reconnection, caused by a local violation of the ideal Ohm's law, can be interpreted in a frame independent way as a local "piercing and merging" of connection hypersufaces that lose their identity only locally, in exactly the same way as magnetic field lines do in the standard 3-D space setting.
We note that, even remaining within the validity of the ideal Ohm's law i.e., without allowing for magnetic reconnection to occur, important questions will need to be investigated: in particular how to generalize the topological properties, such as e.g. field line braiding, that have been investigated within a fixed frame 3-D description to the properties of connection hypersurfaces in 4-D Minkowski space.

References

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