Mean MEF in current sheets

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Summary

We present the study of the spatial structure of the so-called tearing modes in current sheets and the mean electromotive force induced by finite resistivity instability. The dependence of the mean EMF on the magnetic helicity of the large-scale field is obtained.

1 Formulation of the problem

The aim is to calculate the mean electromotive force, $\boldsymbol{\mathcal{E}}$, generated by finite resistivity instabilities of a sheet pinch, studied by [1], but in the limit of constant resistivity and negligible gravity. In such case the formulation of the problem must be somewhat different than in the famous aforementioned paper [1], since without gravity and with uniform resistivity the basic unperturbed state, which by assumption possesses a current sheet, requires the presence of strong gas pressure balancing the Lorentz force. Therefore we formulate the problem as follows. The unperturbed state, denoted by subscript 0 satisfies the following equations

$$\mathbf{u}_0 = 0, \quad \nabla p_0 = \frac{1}{\mu_0}(\nabla\times\mathbf{B}_0)\times\mathbf{B}_0, \quad \frac{\partial \mathbf{B}_0}{\partial t} = \eta\nabla^2\mathbf{B}_0, \tag{1}$$

and $\mathbf{B}_0$, by assumption, possesses a current sheet and has the form

$$\mathbf{B}_0 = B_{0x}(y)\hat{\mathbf{e}}_x + B_{0z}\hat{\mathbf{e}}_z, \tag{2}$$

where B_{0z} = const. Because the growth rates of the tearing modes are very large, the time of growth of the instability is very short in comparison with the magnetic diffusion timescale, δ^2/η, where δ denotes the thickness of the current sheet and hence the basic state can be considered quasi-stationary in the stability analysis. We, therefore, neglect the time dependence of the unperturbed magnetic field distribution $\mathbf{B}_0$ in the following. Furthermore, with the form (2) of the basic field the Lorentz force is potential and the Navier-Stokes equation in (1) is simply an equation for the basic pressure distribution.

2 Results

Introducing perturbations, $\mathbf{B} = \mathbf{B}_0 + \mathbf{b}(y)\exp[i(k_x x + k_z z) + \sigma t]$ and $\mathbf{u}(y)\exp[i(k_x x + k_z z) + \sigma t]$, we arrive at th followinge non-dimensional, linearised equations for perturbation fields

$$b_y'' = \left(\kappa^2 + \sigma\right) b_y - \mathrm{i}\kappa\beta u_y, \tag{3}$$

$$u_y'' = \kappa^2\left(1 + \frac{H^2}{\sigma}\beta^2\right)u_y + \mathrm{i}H^2\left(\beta - \frac{1}{\sigma}\beta''\right)b_y, \tag{4}$$

where $\kappa = k\delta$, $H = B_0\delta/\eta\sqrt{\mu_0\rho_0}$ and

$$\beta(y) = \frac{k_x B_{0x}(y) + k_z B_{0z}}{kB_0}. \tag{5}$$

Considering most unstable cases with $\beta(y) = (\kappa_x/\kappa)\tanh(y)$ it is possible to find asymptotic solutions of the above equations (3)-(4) in the limit $H \gg 1$. This, then, allows to calculate the mean electromotive force, which in the additionl limit of thin current sheet, $\kappa \ll 1$ possesses a dominant y-component of the form

$$\mathcal{E}_y \sim \frac{\mathbf{J}_0 \cdot \mathbf{B}_0}{1 + \mathrm{const}\mathbf{J}_0 \cdot \mathbf{B}_0}. \tag{6}$$

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References

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