

Methods of topological data analysis to compare MHD simulations with observations

I. [Makarenko](#), A. Shukurov, P. Bushby, A. Fletcher, R. Henderson

School of Mathematics and Statistics, Newcastle University, United Kingdom

Summary

We show how to apply methods of topological data analysis to compare and quantify the random fields appearing in MHD simulations and observations. We use the Betti numbers, persistence diagrams, and persistence barcodes to achieve the goal.

Due to recent advances in radio telescope instrumentation a number of wide-field high resolution surveys, especially radio emission from hydrogen atoms, are now becoming available. The data are rich enough to distinguish between different physical effects. We also have constantly improving MHD models that simulate the multiphase interstellar medium. The density and velocity distributions in observations, as well as the results of numerical simulations, appear as random fields with anisotropies and complex topology widely believed to be related to turbulence and outflows from the Galactic disk.

What tools can we use to compare and quantify the complicated topology of the random fields appearing in our observations and simulations? How to explore the degree of similarity of images dominated by random structures? How to confirm or disprove the adequacy of MHD models based on comparison with observations? In this talk we use some methods of topological data analysis to answer these questions.

Over the last decade topological approaches to the analysis of random fields have become more popular ([1]-[3]). The topological invariants called the Betti numbers, the Euler characteristic, which can be written as an alternating sum of the Betti numbers, persistence diagrams, and persistence barcodes for the random fields – all these new and rapidly developing techniques are being applied ([4]-[5]).

This talk includes a brief introduction to the basic concepts of topological data analysis illustrated by simple examples in 1 and 2 dimensions. The most promising and popular topological data analysis techniques for analysing images will be demonstrated using slices from MHD simulations of the interstellar medium. I will also use TDA to compare maps of the interstellar hydrogen distribution in different regions of the Galaxy. Using persistence diagrams and persistence barcodes I try to estimate how realistic the MHD model of the interstellar medium is.

Acknowledgements. This work is supported by the Leverhulme Trust Grant "Observations and models of turbulent flows: a topological approach" (2015-2019), RPG-2014-427.

References

- [1] Adler, R. & Taylor, J.E. (2011) Topological Complexity of Smooth Random Functions: École d'Été de Probabilités de Saint-Flour XXXIX-2009. *Lecture Notes in Mathematics*. Springer Berlin Heidelberg. 122 p.
- [2] Ghrist, R. (2014) Elementary Applied Topology. CreateSpace. 276 p.
- [3] Edelsbrunner, H., Letcher D., & Zomorodian A. (2002) Topological persistence and simplification. *Discrete and Computational Geometry*. 2002 **4**, 511–533.
- [4] Sousbie, T. (2011) The persistent cosmic web and its filamentary structure I. Theory and implementation. *MNRAS*. 2011 **414** (1), 350–383.
- [5] Kramar M., Levanger R., Tithof J., Suri B., Xu M., Paul M., Schatz M. & Mischaikow, K. (2015) Analysis of Kolmogorov Flow and Rayleigh-Bénard Convection using Persistent Homology. ArXiv preprint, arXiv:1505.06168, 2015.