Groundstate magnetic energy vs bending energy of knots and links

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Summary

The groundstate magnetic energy spectra of the first 250 prime knots and 130 prime links are compared with the corresponding spectra obtained by computing the bending energy from data given by the `RIDGERUNNER` tightening algorithm (Ashton *et al.*, 2011). We find a remarkable similarity of power laws that demonstrate that even at fundamental energy level curvature information provides a good estimate for magnetic energy contents of complex structures.

1 Groundstate magnetic energy vs bending energy

The groundstate magnetic energy spectra of the first 250 prime knots and 130 prime links were determined by using earlier analytical results [2] and ropelength data given by the `RIDGERUNNER` tightening algorithm [1]. By normalizing magnetic energy with respect to the tight torus configuration we obtained an extremely simple formula for the non-dimensional magnetic energy $\tilde{m}(K)$ of a knot/link type K in terms of its ropelength λ_K [3], that is

$$\tilde{m}(K) = \left(\frac{\lambda_K}{2\pi}\right)^{4/3} . \tag{1}$$

The corresponding energy spectra are shown in Figure 1, where we see that the best-fit curves follow an almost identical logarithmic law. By assuming that the number of knot types grows exponentially with the topological crossing number $c_{\min}$, this common behaviour can be justified by relating ropelength and crossing number according to

$$\lambda(\#_K) \propto c_{\min}^{3/4} , \tag{2}$$

and more precisely

$$\langle \lambda(\#_K) \rangle_{c_{\min}} \geq 2\pi^{1/4} c_{\min}^{3/4} , \tag{3}$$

where angular brackets denote average over each $c_{\min}$-family.

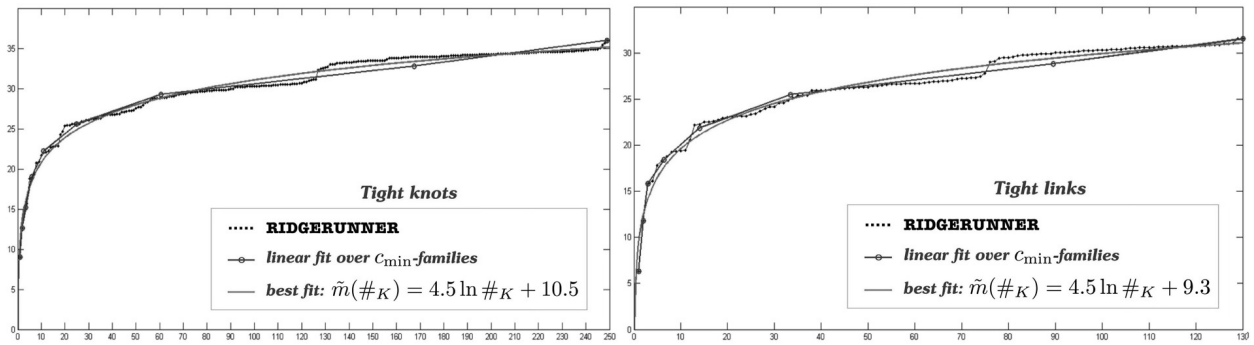

Figure 1: Magnetic energy spectra $\tilde{m} = \tilde{m}(\#_K)$ of the first (a) 250 prime knots and (b) 130 prime links, plotted against the position $\#_K$ of the knot/link listed according to increasing value of ropelength $\lambda_K = \lambda(\#_K)$ (from Ricca & Maggioni, 2013).

2 Comparative analysis between energy spectra of knots and links

Here we compare the groundstate magnetic energy spectra with the spectra obtained by considering the non-dimensional bending energy given by the standard curvature energy divided by that of a tight torus, i.e.

$$\tilde{e}(K) = \frac{\int_K [c(s)]^2 \mathrm{d}s}{2^{4/3}\pi^{5/3}} \ . \tag{4}$$

As suggested by a preliminary analysis [4] both spectra show a remarkable similarity, demonstrating that even at groundstate energy level curvature information provides good estimate for minimum magnetic energy contents. Since curvature is a geometric quantity that can be more easily estimated by direct inspection, this comparative study shows that morphological information can be used to estimate magnetic energy in contexts where direct measurements are more difficult or more expensive to obtain.

References

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