# Finding topological defects in turbulent superfluid flows.

A. Villois[1], D. Proment[1], H. Salman[1] and G. Krstulovic[2]

[1] *School of Mathematics, University of East Anglia, Norwich Research Park, Norwich, UK*

[2] *Laboratoire J.L. Lagrange, UMR7293, Université de la Côte d'Azur, CNRS, Nice. France.*

**Summary**

We study superfluid dynamics by numerical integration of the Gross-Pitaevskii equation. We develop a vortex tracking algorithm that allows for finding with high accuracy the topological defects of an arbitrary configuration. The algorithm is first used to study vortex reconnections in different settings. Then it is used to track and study some topological and statistical properties of a turbulent tangle generated by the evolution of large-scale initial condition.

Superfluids are usually described by the Gross-Pitaevskii equation that in its dimensionless form reads

$$i\frac{\partial \psi}{\partial t} + \nabla^2\psi - |\psi|^2\psi = 0\,, \tag{1}$$

where $\psi(\mathbf{x}, t)$ is a complex wave-function describing the superfluid *order parameter*. The Gross-Pitaevskii equation, also known as the defocusing nonlinear Schrödinger equation, has been largely used to study vortex dynamics and quantum turbulence. By introducing the Madelung transformation $\psi = \sqrt{\rho}\exp(i\phi)$, the equation (1) can be mapped into a set of equations for the density field $\rho(\mathbf{x}, t)$ and the velocity field $\mathbf{v}(\mathbf{x}, t) = 2\nabla\phi(\mathbf{x}, t)$ of a compressible, inviscid, irrotational and barotropic fluid. Although the vorticity $\omega = \nabla \times \mathbf{v}$ is formally zero, at points where the density vanishes, the phase $\phi$ is not defined and vortices may arise as topological line defects. They mutually influence themselves and, correspondingly, the fluid dynamics. These topological defects, called quantum vortices, have a quantised circulation and correspond to zeros of the wave function. Several questions regarding the superfluid vortex dynamics modelled using the Gross-Pitaevskii equation remain nowadays open. For instance, it is not clear how the energy of vortex configurations initially stored at large scales spreads over scales and eventually decay into phonons (sound radiation).

The Gross-Pitaevskii equation is a rich model that naturally contains reconnections and sound (phonon) emission. However, it does not provide explicit information of vortices and such information must be extracted by some tracking algorithm. Using a specifically-designed algorithm to detect topological defects in the complex wave-function $\psi(\mathbf{x}, t)$ [1], we are able to track with high accuracy the vortex line positions all along the dynamics of any initial vortex line configuration. The algorithm can be use for instance to track all vortices in a turbulent tangle as displayed in figure 1.

This algorithm allows us to measure the time evolution of useful physical quantities like the total vortex line length, and curvature and torsion distributions, which may give useful insights to understand how energy is transferred trough the system scales. We first use the algorithm to study vortex reconnections in different geometries. Then we use the algorithm in a turbulent tangle generated by numerical integration of the Gross-Pitaevskii equation with the so-called Taylor-Green initial condition. From it we are able to infer some information about the spectrum of Kelvin waves, that are helicoidal excitations of vortex lines which are believed to be responsible of the energy cascade in the weakly nonlinear limit.

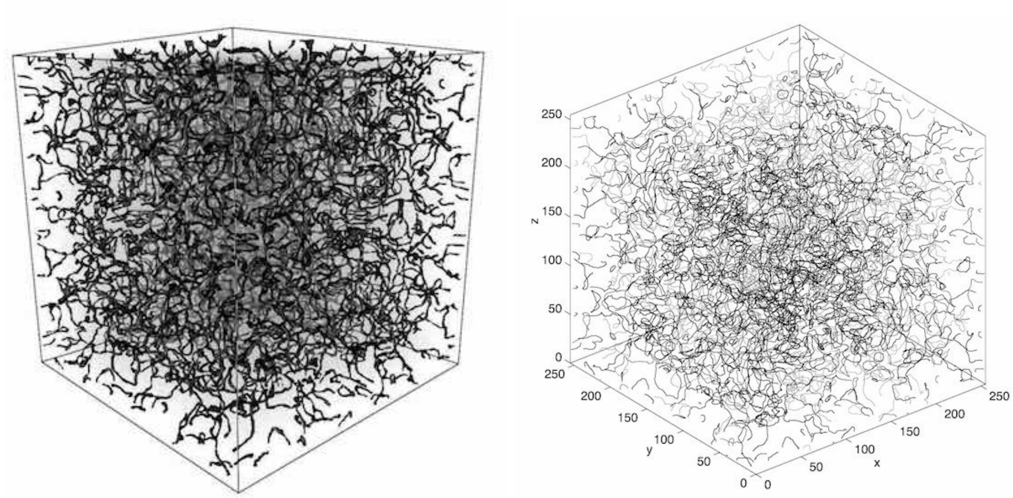


Fig. 1: *Left*: 3D visualisation of the density field. Vortices are represented in blak by an isosurface of the low density regions. *Right*: Corresponding tracking of the vortex tangle. Different tones of gray corresponds to different tracked lines. The turbulent tangle is generated by numerical integration of the Gross-Pitaevskii equation with the so-called Taylor-Green initial condition.

# References

[1] A. Villois, H. Salman, D. Proment, and G. Krstulovic. *An accurate and efficient tracking algorithm to detect topological defects in the Gross-Pitaevskii model.* in preparation.