Geometric generalised Lagrangian mean theories

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Summary

In many applications an ensemble of fluid flows consists of waves riding on a mean flow. Averaging over waves then yields terms in the equation for the mean flow which parameterise the effect of the waves. Theory based on Lagrangian averaging — so called *Generalised Lagrangian Mean* theory — that is tracking the location of fluid parcels, goes back to the works of Soward in 1972 and Andrews & McIntyre in 1978 for *GLM*, and more recently Soward & Roberts 2010 who introduced another averaging method called *glm*. Our goal here is to give a unified approach to these theories using tools from differential geometry and working in the general setting of a flow on a manifold M. As well as the GLM and glm, we introduce Lagrangian means based on geodesics in SDiff(M) as a geometrically sound definition in a general setting. We discuss the advantages and disadvantages of these theories.