Eroding dipoles and vorticity growth for Euler flows in $\mathbb{R}^3$

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Summary

A review of analyses based upon anti-parallel vortex structures suggests that structurally stable vortex structures with eroding circulation may offer a path to the study of rapid vorticity growth in solutions of Euler's equations in $\mathbb{R}^3$. We examine here the possible formation of such a structure in axisymmetric flow without swirl, leading to maximal growth of vorticity as $t^{4/3}$. Our study suggests that the optimizing flow giving the $t^{4/3}$ growth mimics an exact solution of Euler's equations representing an eroding toroidal vortex dipole which locally conserves kinetic energy. The dipole cross-section is a perturbation of the classical Sadovskii dipole having piecewise constant vorticity, which breaks the symmetry of closed streamlines. The structure of this perturbed Sadovskii dipole is analyzed asymptotically at large times, and its predicted properties are verified numerically. The generalization of this flow to an eroding "hairpin" dipole structure in three dimensions is outlined, and proposed as an initial condition favouring a rapid stretching of vortex lines, and possibly a blow-up in finite time. It is argued that the main obstacle to such blow-up is most likely the disruptive effect of axial flow. Our analysis also suggests that conservation of kinetic energy as realized in the eroding hairpin excludes a finite time blow-up for the corresponding Navier-Stokes model.