Topology conserving magnetic field relaxation in plasma

S. Candelaresi[1], D. I. Pontin[1], G. Hornig[1]

[1] *School of Science & Engineering, University of Dundee, UK*

Summary

A method for computing the relaxation of magnetic fields is presented which preserves the field's topology. This method uses Lagrangian grids and mimetic differential operators, which greatly improves the accuracy, as compared to previous approaches. We use this method to study the relaxation of magnetic fields and find that, in absence of magnetic null points, they relax into a force-free state (vanishing Lorentz force) or achieve force-balance with the hydrostatic presure.

Motivated by Parker's hypothesis on the relaxation of topologically non-trivial magnetic braids [1] we develop a method to simulate the ideal (non-resistive) relaxation of magnetic fields with high accurracy. We make use of a Lagrangian grid where the grid points move with the fluid. Together with the ideal induction equation

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{u} \times \mathbf{B}), \tag{1}$$

with the magnetic field $\mathbf{B}$ and fluid velocity $\mathbf{u}$, we can compute the magnetic field at any given time just from the grid distortion $\mathbf{x}$ and initial magnetic field on the initial grid $\mathbf{X}$:

$$\mathbf{x}^*(t)\mathbf{B}(\mathbf{x}, t) = \mathbf{B}(\mathbf{X}, 0), \tag{2}$$

where $\mathbf{x}^*$ is the pull-back. Using this Lagrangian method the topology of the field is conserved at all times.

For the fluid velocity we use the magneto-frictional approach [2]

$$\mathbf{u} = \mathbf{J} \times \mathbf{B}, \tag{3}$$

where $\mathbf{J} = \nabla \times \mathbf{B}$ is the electric current density, which has been shown to be well fit for magnetic relaxation simulations, since it strictly minimizes the magnetic energy. Computing spatial derivatives on a distorted grid can be done directly by referring to the original one [2], but it is shown to be prone to numerical issues [3]. Therefore, we apply mimetic differential operators [4, 5] in order to compute the curl of the magnetic field. It is shown to greatly improve the quality of the field relaxation by several orders of magnitude, depending on the control parameter.

Acknowledgements. We acknowledge the use of the computing facilities HECToR, part of the UK National Supercomputing Service in Edinburgh. All the authors acknowledge financial support from the UKs STFC (grant number STK/K000993/1). We gratefully acknowledge the support of NVIDIA Corporation with the donation of one Tesla K40 GPU used for this research.

References

[1] Parker, E.N. (1972) Topological dissipation and the small-scale fields in turbulent gases. *Astrophys. J.* **174**, 499

[2] Craig, I.J.D. & Sneyd, A.D. (1986) A dynamic relaxation technique for determining the structure and stability of coronal magnetic fields. *Astrophys. J.* **311**, 451-459

[3] Pontin, D.I. & Hornig, G. & Wilmot-Smith A.L. & Craig, I.J.D. (2009) Lagrangian relaxation schemes for calculating force-free magnetic fields. *Astrophys. J.* **700**, 1449

[4] Hyman, J.M. & Shashkov, M. (1997) Natural discretizations for the divergence, gradient, and curl on logically rectangular grids. *Comput. Math. Appl.* **33**, 81-104

[5] Hyman, J.M. & Shashkov, M. (1999) Mimetic discretizations for Maxwell's equations. *J. Comput. Phys.* **151**, 881–909