Helicity on Compact Surfaces

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Consider a magnetic filament or vortex tube anchored at two ends to a compact surface S. The helicity (twist plus writhe) should be conserved if the endpoints do not move, even if the filament is distorted. If the endpoints do move, then the helicity flux will arise from the spins of the two endpoints, plus an orbit term measuring how much the endpoints rotate about each other. Suppose one end is at rest, and the other moves along a path. The moving end has a natural spin (the geodesic curvature) equal to the rotation of the tangent vector to the path relative to parallel transport. The orbit term requires a vector potential. This vector potential can be determined with the help of the Gauss-Bonnet theorem. If space is foliated by a set of nested compact surfaces, then we can ask whether helicity (or relative helicity) can be considered as a sum of 'surface helicity densities'; more precisely an integral of a function on the set of surfaces. If the surfaces are parallel planes or concentric spheres, then this function can be simply described in terms of the linking of poloidal and toroidal fluxes. For less symmetric surfaces, we will generalize the poloidal-toroidal flux decomposition in a manner which allows us to maintain this idea of surface helicity density.