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MYSTERIOUS TRIALITY AND M-THEORY

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ABSTRACT

Mysterious duality was discovered by Iqbal, Neitzke, and Vafa [1] as a convincing, yet mysterious correspondence between certain symmetry patterns in toroidal compactifications of *M*-theory and del Pezzo surfaces, both governed by the root system series E_k. It turns out that the sequence of del Pezzo surfaces is not the only sequence of objects in mathematics which gives rise to the E_k symmetry pattern. I will present a sequence of topological spaces, starting with the four-sphere S^4, and then forming its iterated cyclic loop spaces $L_c^k S^4$, within which we will see the E_k symmetry pattern via rational homotopy theory. For this sequence of spaces, the correspondence between its E_k symmetry pattern and that of toroidal compactifications of *M*-theory is no longer a mystery, as each space $L_c^k S^4$ is naturally related to the compactification of *M*-theory on the *k*-torus via identification of the equations of motion of (11-*k*)-dimensional supergravity as the defining equations of the Sullivan minimal model of $L_c^k S^4$. This gives an explicit duality between rational homotopy theory and physics. Thereby, Iqbal, Neitzke, and Vafa's mysterious duality between algebraic geometry and physics is extended to a triality involving algebraic topology, with the duality between topology and physics demystified. The mystery is now transferred to the mathematical realm as duality between algebraic geometry and topology.

This is a joint work [2] *with Hisham Sati.*

[1] Iqbal, A., Neitzke, A., Vafa, C. 2002 A mysterious duality. *Adv. Theor. Math. Phys*. **5**, 769-808. arXiv:0111068 [hep-th].

[2] Sati, H., Voronov, A.A. 2021 *Mysterious Triality*. Preprint IPMU21-0084. arXiv:2111.14810 [hep-th].