MAGNETIC HELICITY IN MULTIPLY CONNECTED DOMAINS AND THE PROOF OF TAYLOR'S CONJECTURE

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ABSTRACT

Magnetic helicity is a topological invariant of ideal magnetohydrodynamics that describes the linkage of field lines weighted by magnetic flux. In simply connected and magnetically closed domains, its definition is the integral, over the domain, of the dot product of the magnetic field with a vector potential of the magnetic field. This definition is gauge-invariant, i.e. it does not depend on the choice of the vector potential. Often, however, it is necessary to consider magnetic helicity in multiply connected domains, such as a torus or a toroidal shell – two domains important in plasma physics. The standard definition of magnetic helicity in simply connected domains for any vector potential is no longer applicable in multiply connected domains as it is not gauge invariant. In this contribution, we present two main results. (1) The extension of the definition of magnetic helicity to bounded and multiply connected domains, making the definition gauge invariant. (2) The use of this extended definition to complete the landmark proof of Taylor's conjecture in [1], allowing for arbitrary vector potentials to be used in multiply connected domains. Further details can be found in [2] and [3].

This is joint work with Daniel Faraco, Sauli Lindberg and Alberto Valli.

[1] Faraco, D., Lindberg, S. 2020 Proof of Taylor's conjecture on magnetic helicity conservation. *Comm. Math. Phys.* **373**, 707-738.
[2] MacTaggart, D., Valli, A. 2019 Magnetic helicity in multiply connected domains, *J. Plasma Phys.* **85**, 77580501.
[3] Faraco, D., Lindberg, S, MacTaggart, D., Valli, A. 2022 On the proof of Taylor's conjecture in multiply connected domains. *Appl. Math. Lett.* **124**, 107654.