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RECONNECTION NUMBER OF VORTEX KNOTS

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ABSTRACT

Knotted vortices such as those produced by Kleckner and Irvine at the Frank Institute of the University of Chicago tend to transform by reconnection to collections of unknotted and unlinked circles [1], [2]. The reconnection number R(K) of an oriented knot of link K is the least number of reconnections (oriented re-smoothings) needed to unknot/unlink K. Putting this problem into the context of knot cobordism, we show, using Rasmussen's Theorem ([3], [4], [5]) that the reconnection number of a positive knot is equal to twice the genus of its Seifert spanning surface. In particular an (a,b) torus knot has R = (a-1)(b-1). For any knot or link diagram K, we show that R(K) is counted from above by c(K) - s(K) + 1 where c(K) is the number of crossings of K and s(K) is the number of Seifert circles of K. For an arbitrary positive knot or link K, this is an equality. That is, for K a positive link, R(K) = c(K) -s(K) + 1. Examples of vortex dynamics will be illustrated.

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