

STABLE KNOTS AND LINKS IN ELECTROMAGNETIC FIELDS

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ABSTRACT

An electromagnetic field consists of two time-dependent vector fields \mathbf{E}_t and \mathbf{B}_t that satisfy Maxwell's equations. At any fixed time t closed field lines of \mathbf{E}_t or \mathbf{B}_t form knots and links. If an electromagnetic field is *null* for all time, the time evolution of its field lines is given by a 1-parameter family of diffeomorphisms. In particular, the knot types of the closed field lines do not change with time. They are *stable*.

The most famous example of a null field with knotted field lines is Rañada's Hopfion, where all field lines are closed and any pair of them forms a Hopf link [4]. In 2013, Kedia, Bialynicki-Birula, Peralta-Salas and Irvine used a construction by Bateman [1] to obtain any torus knot as a closed field line of a null electromagnetic field [3].

In this talk I will use Bateman's construction and its relation to contact topology to prove that every knot and every link arises as a set of closed field lines of a null electromagnetic field [2].

- [1] Bateman, H. 1915 *The mathematical analysis of electrical and optical wave-motion*. Dover.
- [2] Bode, B. 2021 Stable knots and links in electromagnetic fields. *Commun. Math. Phys.* **387**, 1757-1770.
- [3] Kedia, H., Bialynicki-Birula, I., Peralta-Salas, D., Irvine, W.T.M. 2013 Tying knots in light. *Phys. Rev. Lett.* **111**, 150404.
- [4] Rañada, A.F. 1989 A topological theory of the electromagnetic field. *Letters in Mathematical Physics* **18**, 97-106.