

PROOF OF ZERO-HELICITY FOR THE GROSS-PITAEVSKII EQUATION BY NOETHER THEOREM

ANDREA BELLONI

Department of Mathematics & Applications, U. Milano-Bicocca, Italy

E-mail: a.belloni17@campus.unimib.it

ABSTRACT

In ideal conditions the standard kinetic helicity, that is well-defined for classical fluid systems, is a well-known conserved quantity. In quantum fluids its rigorous definition needs an appropriate adjustment due to the lack of regularity for the presence of defects. As shown by Salman [1], and subsequently proven using topological arguments by Sumners *et al.* [2], helicity can be demonstrated to be invariably zero for systems that admit Seifert framing. A typical example of such systems is given by quantum vortex defects governed by the Gross-Pitaevskii equation. In this case an attempted proof of zero-helicity using Noether theorem was given by Kedia *et al.* [3]. Their demonstration, however, has several inconsistencies. Here by making use of Kleinert's [4] multi-valued gauge theory and Seifert surface properties we provide a rigorous proof of this result.

This is joint work with Renzo Ricca.

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- [3] Kedia, H., Kleckner, D. Scheeler, M.W. and Irvine W.T.M. 2018 Helicity in superfluids: existence and the classical limit. *Phys. Rev. Fluids* **3**, 104702.
- [4] Kleinert, H. 2008 *Multivalued Fields in Condensed Matter, Electromagnetism and Gravitation*. World Scientific, Singapore.